PROJECT GUSH II

INVESTIGATION OF PRECIPITATION PROCESSES
SEMIANNUAL TECHNICAL SUMMARY REPORT

Prepared for:

ADVANCED RESEARCH PROJECTS AGENCY
DEPARTMENT OF DEFENSE
WASHINGTON 25, D.C.

R.R. Rogers
Contract No. Nonr 3672(00)
ARPA Order No. 263
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31 January 1963

CORNELL AERONAUTICAL LABORATORY, INC.
OF CORNELL UNIVERSITY, BUFFALO 21, N.Y.

CORNELL AERONAUTICAL LABORATORY, INC.
BUFFALO 21, NEW YORK

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FOR PERIOD ENDING

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PROJECT GUSH II
Investigation of Precipitation Processes

SEMIANNUAL TECHNICAL SUMMARY REPORT

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INTRODUCTION

Under Contract Nonr 3672(00) with the Office of Naval Research, Cornell Aeronautical Laboratory is conducting a program to investigate precipitation processes. The program is divided into two phases and includes: (1) studies of cloud and precipitation processes using pulsed Doppler radar, and (2) measurements of atmospheric space-charge in the vicinity of clouds. This is a summary of the technical activities under item 1 above during the first half of the program, which ended 31 January 1963.

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DOPPLER RADAR INVESTIGATIONS

The purpose of this phase of the project is to use a Doppler radar system for quantitative measurements of characteristics of the wind field and the hydrometeors associated with precipitation processes in the atmosphere. It was demonstrated in an earlier project (Rogers and Pilié, 1962) that radar techniques could be used for measuring drop-size distribution in rainfall under stable atmospheric conditions. It is intended to extend these techniques in the present project to provide measurements of the growth rate of rain drops that occur in relatively stable atmospheric conditions and to make simultaneous measurements of drop-size distributions and the speed of updrafts in convective clouds[1]. Further we plan to investigate the relationship between turbulence and the precipitation process.

At the time this program was initiated the relatively insensitive Doppler radar used for the Mt. Withington experiments (Rogers and Pilié, 1962) was planned for use on the project. Shortly thereafter the Laboratory provided funds for extensive modification and improvement of the radar. The modification program encountered difficulties which as indicated by Section B of this report are only now being ironed out. At the present time we are confident that the radar will be in operation in the field by approximately 15 March.

During the time the equipment has been inoperative we have completed the analysis of certain problems which arise in interpreting Doppler radar data from weather targets. This analytical work has led to two papers which are attached as appendices. One of the papers will be published in the March issue of the Journal of the Atmospheric Sciences and the other was presented last October at the University of Michigan's Second Conference on Remote Sensing of Environment. The conclusions of both these papers are summarized

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[1] Since the inception of this program, a report has been given of such simultaneous measurements of updraft intensity and size distributions. (Battan, L. J., et al.; "Observations of showers by means of pulse Doppler radar," paper delivered at 211th Meeting of Amer. Meteor. Soc., Jan. 1963, New York).

in the body of this report in addition to an analysis of the interaction between hydrometeors and the wind, which is essential for determining how accurately the wind can be measured with Doppler radar. Finally, a brief description will be given of some of the characteristics of the improved Doppler radar system.

A. Analytical Work

1. Meteorological information contained in radar signal fluctuations[2]

The most complete information obtainable from fluctuations in a weather radar signal is the distribution, within the sampled volume, of the radial component of the velocities of the hydrometeors responsible for scattering the electromagnetic energy. Coherent Doppler radar is needed to obtain this velocity distribution, from which properties of the target such as drop sizes and wind speed can be inferred. It has been known for some time that less detailed information can be derived from measurements with ordinary incoherent pulsed radars. A theory for interpreting incoherent signals was worked out by Fleisher (1953) on the basis of an analysis by A.J.F.Siegert, which is reported by Lawson and Uhlenbeck (1950). Siegert's analysis can also be used to explain the relationship between the power spectrum of the signal received by a Doppler radar and the velocities of the scatterers. This relationship then provides a basis for explaining coherent or incoherent measurements.

The fundamental relationship between the measured power spectrum $F_m(f)$ of the received signal and the radial velocity distribution $S(u)$ of the scatterers is found to be

$$F_m(f) = S\left(u - \frac{f'\lambda}{2}\right) \tag{1}$$

where f' is the frequency to which the received signal may be heterodyned for convenience in spectrum analysis. $F_m(f)$ and therefore the velocity distribution are available in Doppler radar experiments.

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[2] A detailed treatment of this topic is given in Appendix A.

If incoherent radar is used instead of Doppler radar, the measurable property of the signal is its power. The spectrum of the power fluctuations is determined by the velocity distribution, but in general the relationship is not unique so that this spectrum can not be used to obtain the velocity distribution. Only in the special case of a symmetrical distribution of velocity can measurements of signal power yield the distribution. The rate at which the fluctuating power crosses its average level, however, affords a simple measurement of the spread of the velocity distribution, regardless of its exact shape. If the rate of average power level crossings is L, then

$$L = 5.22 \; \sigma_u / \lambda \tag{2}$$

where σ_u is the standard deviation of the radial component of velocity and λ is the radar wavelength.

2. Interpretation of the velocity distribution

The basic piece of information available from Doppler radar measurements is the velocity distribution $S(u)$ of the hydrometeors in the sampled volume. This velocity distribution is determined by the wind in the sampled volume, the response of the hydrometeors to the wind, and their motion relative to the air due to gravitational settling. In the case of vertical viewing in rain and in the absence of significant updrafts, the velocity distribution is caused entirely by gravitational settling. Then the drop-size distribution $n(D)$ is related to the measured velocity distribution by

$$n(D) \propto \frac{S(u)}{D^6} \frac{du}{dD} \; , \tag{3}$$

where $u(D)$ is the terminal fall velocity of a drop having diameter D (cf. Rogers and Pilié, 1962).

When turbulence and updrafts are present, the complete drop-size distribution cannot be computed from the spectrum. In this situation it is possible, however, to obtain an estimate of both the speed of the updrafts and the size of the largest drops present[1]. The low-velocity end of the spectrum is attributed to the very small drops which move essentially with the wind. The high-velocity end is attributed to the large drops. Since the vertical speed of the wind and the downward speed of the large drops toward the radar are both measured, the size of the drops can be computed assuming they fall with terminal velocity relative to the air.

When the radar is pointed horizontally, the velocity component due to gravitation is not observed, and all the velocities in the distribution can be explained in terms of the horizontal wind and the response of the hydrometeors to this wind. Likewise, for any viewing angle, such weather targets as clouds and snow have velocities containing only small contributions from gravitation. In order to derive information about the wind from the Doppler spectra from these targets requires understanding the way the hydrometeors respond to the wind.

3. Response of hydrometeors to wind

If raindrops and snowflakes move exactly with the wind, then their Doppler spectra represent the distribution of the radial component of wind in the sampled volume. The mean velocity in the spectrum would correspond to the mean wind in the volume and the spread of the spectrum would provide a measure of turbulence and shear within the sampled volume. This interpretation of the mean velocity was reported by us previously[3] and has also been used by Lhermitte and Atlas (1961). A small error could be introduced into these mean wind measurements if strong shear is present. In that case, a raindrop falling into the sampled region would not immediately assume the velocity of the air in its environment, but would fall some distance before establishing equilibrium. These effects in measurements of mean velocity would usually be quite small.

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[3] Project GUSH Occasional Report, Nov. 1961.

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A more difficult problem is to describe the manner in which quantitative descriptions of turbulence can be obtained from Doppler measurements. For this application it is necessary to know how well hydrometeors respond to the rapid turbulent fluctuations in wind. The forces acting on a hydrometeor are assumed to be gravitation, buoyancy, and drag. Of these, only the non-linear drag force is troublesome. The drag is given by $\frac{1}{2} C_D A \rho_a V_r^2$ where C_D is the drag coefficient, ρ_a is the density of ambient air, A is the cross sectional area of the hydrometeor and V_r is the relative velocity between hydrometeor and air. Two kinds of weather targets, which in a sense represent limiting cases, can easily be treated because their pertinent equations of motion can be linearized.

The first of these special cases is a target consiting of cloud droplets. When the Reynolds number is less than approximately 10, the drag coefficient for a sphere varies inversely with velocity. In this region of flow, the drag force on a sphere of diameter D is equal to $3\pi\mu V_r D$ where μ is the viscosity of air. Because of the limitation on Reynolds number this form for the drag applies only to droplets smaller than about 0.1 mm in diameter, which includes almost all cloud droplets. The equation of motion for one of these small drops is

$$m \frac{d\vec{V}}{dt} = 3\pi\mu D \vec{V_r} - m\vec{g} + m_a \vec{g} .$$

In order, the terms on the right are the drag force, gravitational force, and buoyancy force. Since the mass m_a of air displaced by the droplet is approximately 10^{-3} times the mass m of the droplet, the buoyancy term can be neglected. Then the equation reduces to

$$\frac{d\vec{V}}{dt} = k_1 \vec{V_r} - \vec{g}$$

where $k_1 = 3\pi\mu D/m$. Substituting $\vec{V_r} = \vec{V_m} - \vec{V}$, where $\vec{V_m}$ is the velocity of ambient air, yields

$$\frac{d\vec{V}}{dt} + k_1 \vec{V} = k_1 \vec{V_m} - \vec{g} .$$

The equation for vertical components is

$$\frac{dv_z}{dt} + k_1 V_z = k_1 V_{mz} - g,$$

with obvious notation. In the case of stagnant air $V_{mz} = 0$; then the steady state solution is simply

$$V_z = -\frac{g}{k_1} = V_t,$$

the terminal fall velocity.

The general steady state solution for the case in which the air oscillates sinusoidally, according to $V_{mz} = w e^{i\omega t}$, is

$$V_z = V_t + \frac{k_1 w}{k_1 + i\omega} e^{i\omega t}.$$

This result can be expressed in terms of filter theory: the input or driving function is V_{mz} while the output is the drop's response V_z. The ratio of output (adjusted by the terminal fall velocity) to input is

$$\frac{V_z - V_t}{V_{mz}} = \frac{1}{1 + i\frac{\omega}{k_1}} \equiv \ell(\omega),$$

which is identical to the response function for a low pass filter. The analogy can be carried further with useful results by computing the system function, which by definition is

$$H(\omega) \equiv \ell(\omega) \ell^*(\omega).$$

For this case

$$H(\omega) = \frac{1}{1 + \left(\frac{\omega}{k_1}\right)^2}. \tag{4}$$

The usefulness of the system function lies in its property that the spectrum of the output is the spectrum of the input multiplied by $H(\omega)$.

The system function corresponding to horizontal motions can be shown to be identical to (4). Substituting for k_1, and setting $\mu = 1.7 \times 10^{-4}$ in cgs units leads to

$$H(\omega) = \frac{1}{1 + 1.07 \times 10^5 \omega^2 D^4}, \quad D \text{ in } cm,$$

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as the system function in terms of droplet size, for diameters smaller than 0.1 mm. Figure 1 shows this function for a droplet of 0.1 mm diameter which corresponds to the largest cloud droplets and the largest permissable size for this approximation. It is seen in the figure that frequencies as high as one cycle per second are followed to within 95% by the droplet.

A second special case for which the equation of motion can be linearized, is applicable to the vertical component of a drop having high terminal fall velocity, and was first recognized by Stackpole (1961).

The general equation of vertical motion for raindrops having Reynolds numbers higher than about 300 is

$$\frac{dV_z}{dt} = k_2 |V_r| V_r - g$$

where $k_2 = C_D \rho_a \pi D^2 / 8m$ and $V_r = V_{az} - V_z$. This form is appropriate for drops larger than 1 mm. It is assumed that the main contribution to V_r is the terminal fall velocity V_t. Turbulent deviations are then taken to be perturbation quantities, negligible in higher orders. The system function is then found to be

$$H(\omega) = \frac{1}{1 + \left(\frac{\omega V_t}{2g}\right)^2}.$$

It is more convenient to have the system function in terms of drop diameter rather than terminal velocity. The data of Gunn and Kinzer (1949) can be used for this conversion. The system function for various drops is shown in Figure 2.

One concludes from the results shown in Figs. 1 and 2 that all cloud droplets respond satisfactorily to be considered good tracers for gusts, but that large raindrops do not. For example, the 5 mm drop is seen to have a spectral response of only 10% to a 1 cps driving force, a value entirely inadequate for a tracer material.
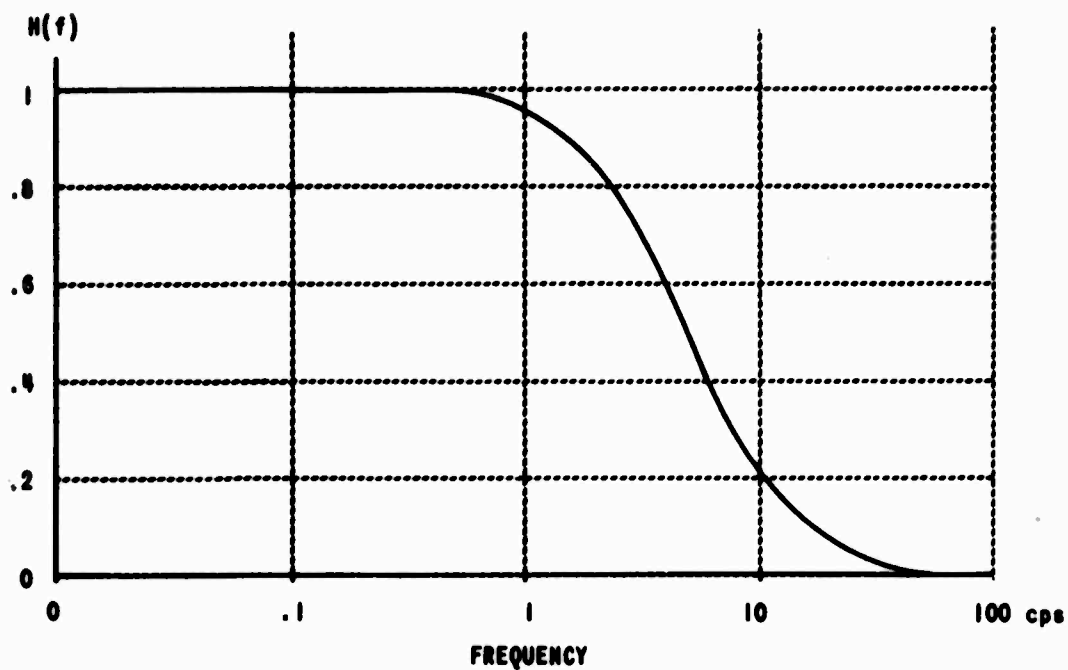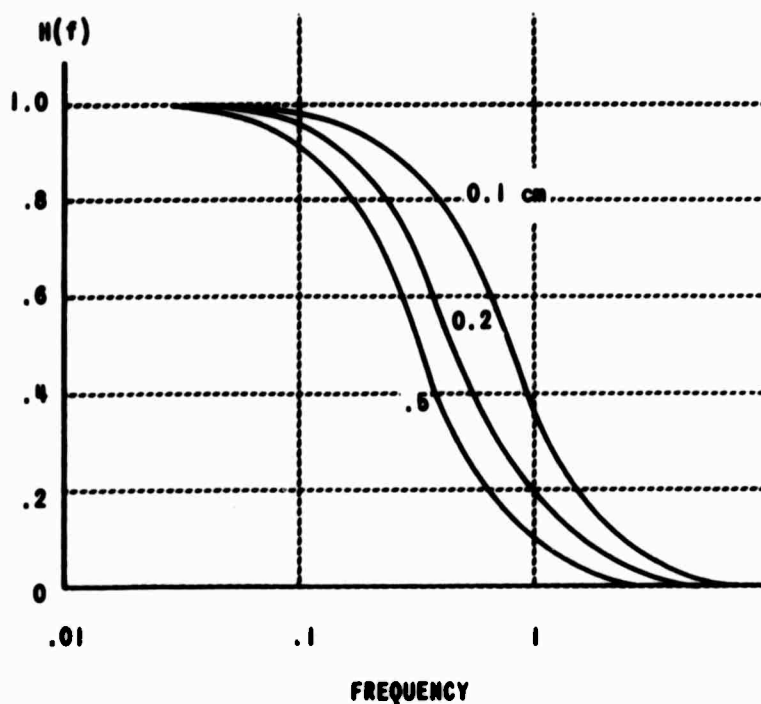
Figure 1 RESPONSE FUNCTION FOR 0.1 mm CLOUD DROPLET

Figure 2 RESPONSE FUNCTIONS FOR FALLING RAINDROPS

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Still unaccounted for are the drops too large for the Stokes flow approximation and too small for the perturbation approximation. These are drops in the approximate interval 0.1 - 1 mm. The equation of motion must be solved in its non-linear form in this region. Further, since the equation in non-linear, the concept of a system function does not apply. Numerical solutions to the equation carried out on an analog computer indicate that only the smallest of the drops in the 0.1 - 1 mm range can be considered adequate tracers for moderate wind gusts.

This same non-linear analysis must be applied to snowflakes, as their equations of motion do not lend themselves to linearization. The conclusion of the analysis is that dry snowflakes are satisfactory tracers for gusts, but that large wet snowflakes respond too slowly to give a good representation of the wind.

In conclusion, the analysis of hydrometeor response has shown that targets consisting of dry snowflakes or cloud particles are nearly perfect indicators of all scales of atmospheric turbulence. Wet snow and rain, however, respond poorly to high frequency gusts and can only be used as indicators of scales of motion characterized by low frequencies, as for example, the "mean wind" of scales comparable to those sensed by a radio-sonde balloon.

4. Measurement of turbulent energy[4]

For those weather targets consisting of particles that respond faithfully to the wind, the velocity distribution of the particles can be interpreted as that of the wind. Therefore, Doppler radar can be considered as a volume-sampling anemometer. As well as providing an estimate of the mean wind, the Doppler measurements can be analyzed to obtain the total energy in the radial component of the turbulent wind. It is required that two properties of the Doppler spectrum be recorded continually: the mean frequency (or mean radial velocity) and the variance of the frequency, as indicated by the Doppler spectrum. These quantities are denoted respectively by $\langle V \rangle$ and σ_V^2.

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[4] This subject is treated in more detail in Appendix B.

Both of these quantities can be measured in real time using analog techniques. The average velocity is an indicator of relatively large scales of turbulence, while the variance is an indicator of the relatively small scales. The total energy in all scales of turbulence can be obtained by analyzing both of these quantities. In particular, let $\sigma^2_{\langle v \rangle}$ denote the variance of $\langle V \rangle$ over several minutes of record; and let $\overline{\sigma^2_v}$ denote the average value of σ^2_v over the same period. Then the energy E, per unit mass of air, in the measured component V of turbulence is related to these quantities by

$$E = \frac{1}{2}\left[\sigma^2_{\langle v \rangle} + \overline{\sigma^2_v}\right]. \tag{5}$$

B. Radar Equipment

1. Introduction

A detailed investigation was made of the CAL Doppler radar at the beginning of this project[5]. This investigation indicated that several modifications and additions to the system would be desirable for meteorological observations such as planned on this program. Briefly these include:

a. The installation of a high quality stalo in order to improve the short-term frequency stability of the equipment and hence the Doppler velocity resolution capability,

b. The installation of stablized amplifiers and bias power supplies in order to provide accurate and stable echo cross-section measuring capability,

c. Improved transmitter pulse shape and peak power output,

d. Elimination of the reference signal in the Doppler spectra by redesign of the receiver,

e. Improvement of receiver dynamic range and linearity characteristics,

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[5] This investigation is documented in a CAL memorandum: B.R. Tripp, MEMORANDUM FOR THE RECORD, "Summary Report of DOP Radar Investigation and Proposed Improvements," 13 August 1962, BRT:pwm-26.

f. Addition of an average frequency tracker,

g. Elimination of spectra aliasing problems by altering the bandpass characteristics of the receiver, and

h. Other more minor but important modifications to correct drift and spurious responses in the receiver.

The following sections discuss briefly major aspects of the radar system and related instrumentation now available for this project.

2. System Design

The original Doppler radar system produced a strong reference signal component in all spectra. This signal was caused by signal leakage from the reference oscillator heterodyning with the local oscillator signal. Normally this signal component is of no concern. However, in order to accurately measure the mean frequency of Doppler shifted spectra, it was necessary to eliminate or suppress this leakage components to below the receiver noise level. A redesign of the system has produced an acceptable feedthrough level on the basis of early bench tests of the modified receiver. Final performance will be measured after the equipment is reinstalled in the radar van. A block diagram of the modified receiver and transmitter (block diagram) is given in Figure 3.

The second major problem was the short-term frequency stability of the receiver stalo. The original radar used a free-running reflex klystron oscillator that was unstable and hence unsuitable for accurate measurement of Doppler spectra. This problem has been corrected by installing a stalo with a short term frequency stability of 1 part in 10^8, or 100 cps maximum drift at X-band. During time intervals corresponding to the anticipated signal propagation time, the frequency drift should be one order or more of magnitude less than the maximum permissible value of 100 cps. An optimistic estimate of Doppler frequency resolution of the system is 10 cps for target ranges of a few kilometers.

Figure 3 BLOCK DIAGRAM OF THE CAL PULSE DOPPLER RADAR
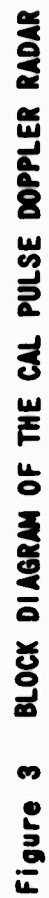
3. Receiver Characteristics

Basically, the receiver design has been altered to reduce spurious responses and to extend its dynamic range. Several very narrow bandwidth filters have been added prior to various heterodyning processes to minimize aliasing errors in the Doppler spectra. All spectra foldover products are down by at least 55 db in the Doppler output signal.

A typical frequency response curve of the modified receiver is given in Figure 4. This curve shows the receiver response over a 10 kc/s total bandwidth. The slight asymmetry is attributed to the response of a mechanical filter which is included in the system in order to achieve the rapid fall-off of frequency response outside the receiver bandpass. The effect of asymmetry can be removed if necessary from the Doppler signal by multiplying the Doppler spectra by the inverse of the system frequency response.

A sample calibration curve of receiver linearity is given in Figure 5. This curve illustrates the wide dynamic range capability of the modified Doppler receiver.

4. Auxiliary Instrumentation

The mean frequency of the Doppler spectra will be automatically measured and recorded with a mean frequency tracker. This instrument is currently being constructed and should be available for tests within the next few weeks.

Other instrumentation which have been added to the radar for this program include:

a. A power detector and pulse integrator for measuring target cross section,

b. Detector for R-meter tests,

c. Calibrated readouts on all system controls, and
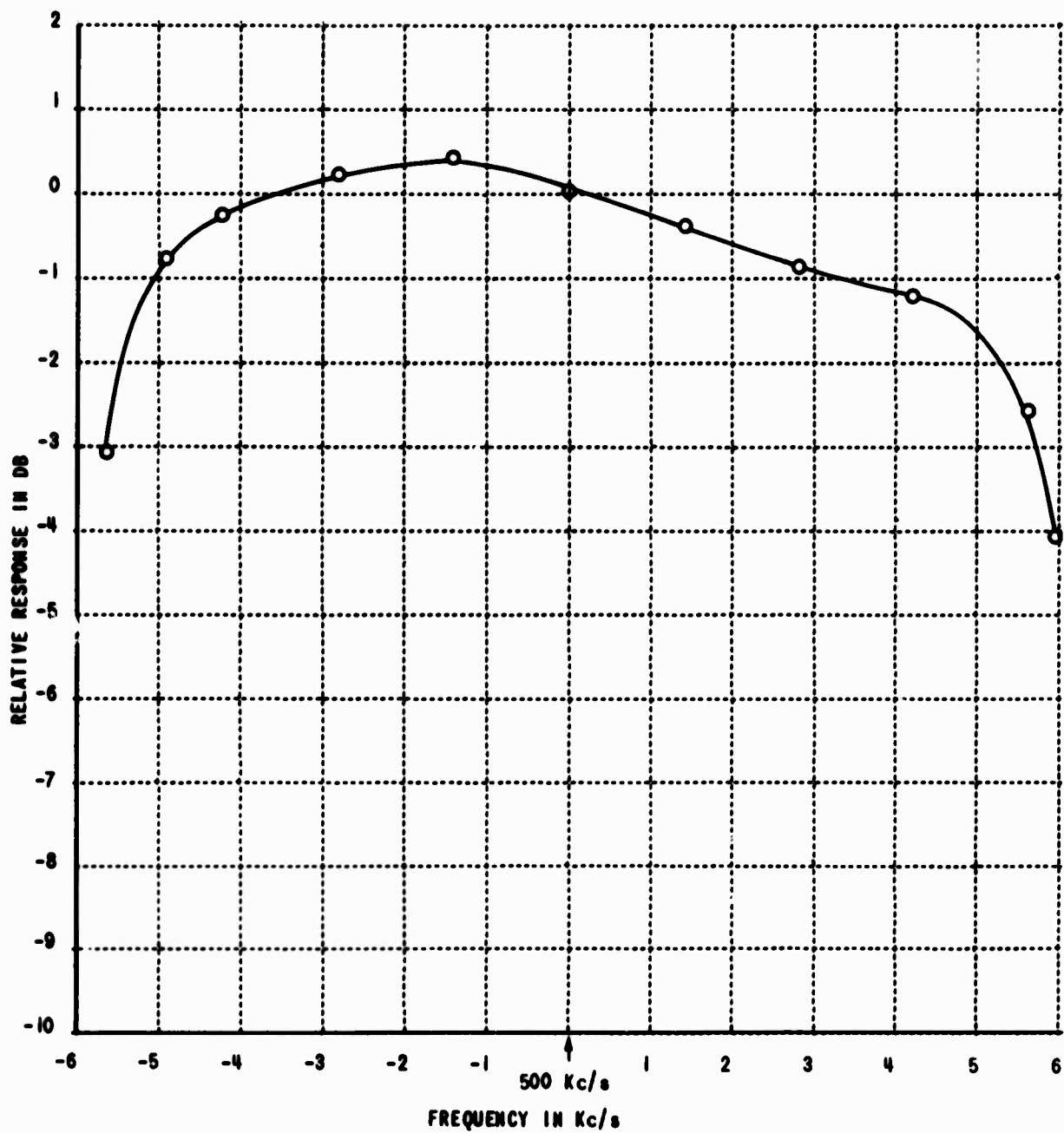
d. Recording facilities for making records of all data.

Figure 4 DOPPLER RECEIVER RESPONSE - 12 Kc/s
FILTER POSITION

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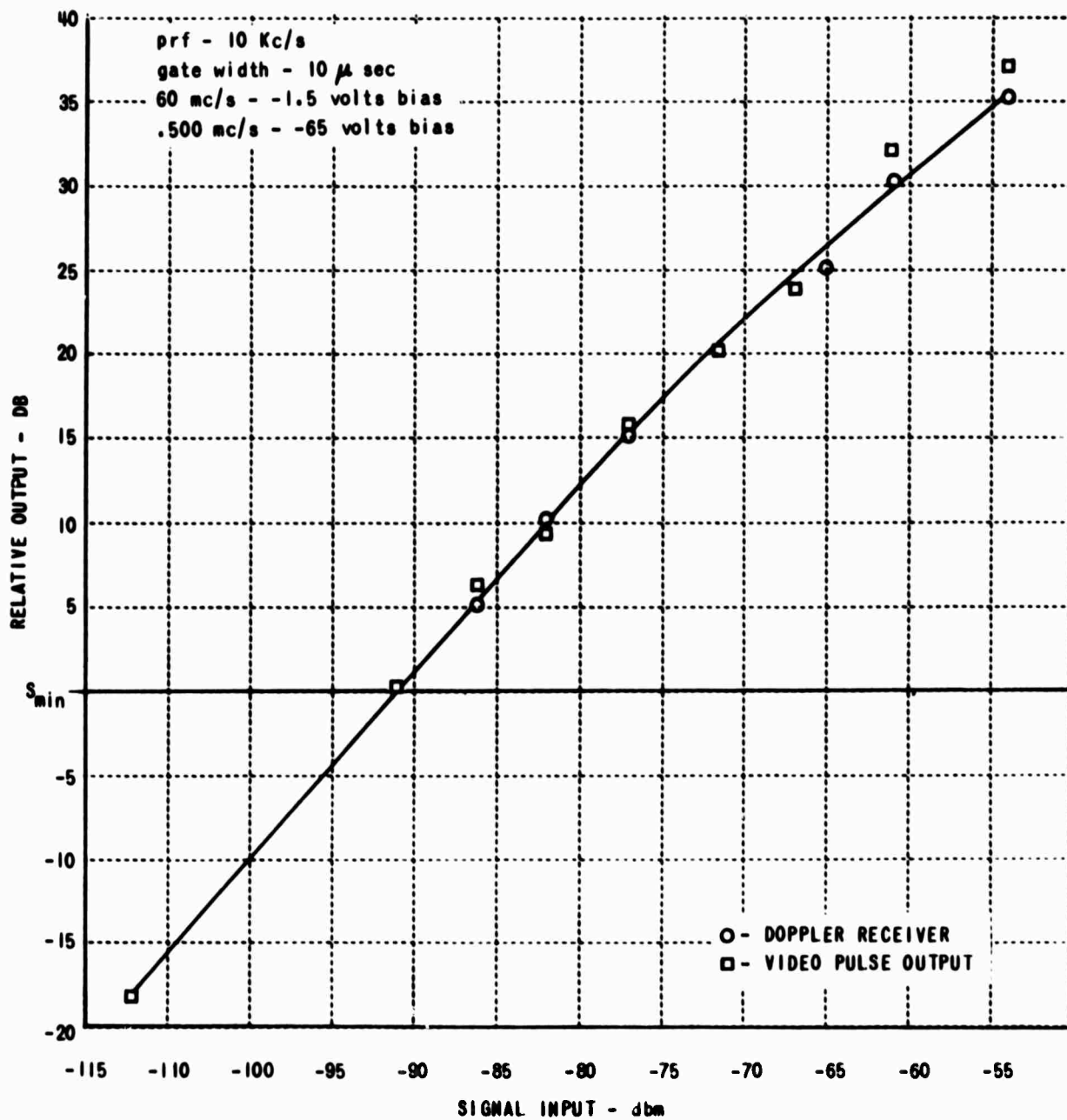
Figure 5   RECEIVER LINEARITY CURVE

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APPENDIX A

RADAR MEASUREMENT OF VELOCITIES

OF METEOROLOGICAL SCATTERERS

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RADAR MEASUREMENT OF VELOCITIES
OF METEOROLOGICAL SCATTERERS[1]

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ABSTRACT

Statistical properties of the field backscattered from a weather radar target are related to the velocity distribution of the scatterers comprising the target. The complete distribution of radial velocity components can be derived from the received radar signal using coherent Doppler techniques. The standard deviation, and in special cases the complete form of this distribution, can be obtained from incoherent radar measurements.

1. Introdu.ion

    When the raindrops or snoflakes comprising a weather radar target
move toward or away from the transmitter, the backscattered field, observed
at a point, is seen to fluctuate randomly in both amplitude and phase.
It is well known that these fluctuations can be analyzed to obtain certain
information about the velocities of the scatterers. The most complete
information obtainable is the distribution of the radial components of
the velocities of the scatterers. To measure this veloci., distribution
requires the use of carefully designed Doppler radar equipment. Such
measurements have been interpreted in terms of the mean wind profile
(Lhermitte and Atlas, 1961; Lhermitte, 1962), updrafts in showers
(Probert-Jones and Harper, 1961), and raindrop-size distributions (Boyenval,
1960; Probert-Jones, 1960; Rogers and Piliē, 1962).

    Before Doppler radars became available for meteorological research,
incoherent radars were used to measure some properties of the velocity
distributions in weather targets. The complete velocity distribution cannot
be derived from these measurements, in general, because the phase
information is lost without coherent (Doppler) techniques. In past years,
however, considerable attention was given to measuring wind shear and
turbulence with incoherent radars; these efforts are reviewed by Battan (1959).
The theoretical foundation for interpreting incoherent radar measurements
was worked out by Fleisher (1953) on the basis of the theory of A.J.F.Siegert,
which is reported for example by Lawson and Uhlenbeck (1950).

    The analysis of radar signals to determine the information about
velocities contained therein can be based on Siegert's theory, whether
incoherent or coherent radar is used. In view of the increasing use of
Doppler radar in meteorological work, it may be helpiul to show how the
interpretation of Doppler measurements can be justified and to show the
connection between Doppler and incoherent measurements.

2. The autocorrelation of the complex amplitude

The backscattered field from the moving collection of hydrometeors constituting a weather radar target will, in general, be both amplitude- and phase-modulated. A Cartesian component of its electric vector at a fixed point in space can therefore be represented as

$$E(t) = A(t) \cos\left[\phi(t) - \omega_o t\right], \qquad (1)$$

where $\omega_o = 2\pi f_o$ is the angular frequency of the carrier wave, and $A(t)$ and $\phi(t)$ are real functions demonstrating the amplitude and phase modulations. An equivalent representation of $E(t)$ is

$$E(t) = \mathcal{R}\left[A(t) e^{i(\phi(t) - \omega_o t)}\right],$$

where $\mathcal{R}$ denotes the real part. This representation is useful because exponential functions are often easier to manipulate in calculations than trigonometric functions.

A third representation, and the one we will use to describe the signal, is

$$E(t) = \mathcal{R}\left[e(t) e^{-i\omega_o t}\right]. \qquad (2)$$

The quantity $e(t) e^{-i\omega_o t}$ is called the analytic signal associated with the real signal $E(t)$. The complex amplitude $e(t)$ is given by

$$e(t) = e_r(t) + i\, e_i(t) = A(t) e^{i\phi(t)},$$

and contains both the amplitude and phase modulation effects. It is made up of contributions from all scatterers comprising the target:

$$e(t) = \sum_{k} r_k(t) e^{i\phi_k(t)},$$

where r_k and ϕ_k are the amplitude and phase of the component from the kth scatterer. This complex amplitude is not a measured quantity, but merely a mathematical convenience. Certain properties of $e(t)$ are closely related to corresponding properties of the real signal, however, and computations are more readily carried out on the analytic signal.

As the scatterers move about, the amplitudes of their individual contributions to the composite field are assumed to be constant while the phases vary according to

$$\phi_k(t) = \frac{4\pi}{\lambda} u_k t ,$$

where λ is the radar wavelength and u_k is the velocity component of the kth scatterer in the direction of the transmitter. Using this model for the scattered field, Siegert derived the autocorrelation function $G(\tau)$ of the complex amplitude:

$$G(\tau) = \frac{\overline{e(t)e^*(t+\tau)}}{\overline{|e(t)|^2}} = \frac{1}{\overline{r^2}} \int_0^\infty r^2 dr \int_{-\infty}^\infty W(r,u) e^{4\pi i u \tau / \lambda} du, \qquad (3)$$

where overbars denote ensemble averages and asterisks denote complex conjugates. The average quantity $\overline{r^2}$ is proportional to the power in $E(t)$ and is defined by

$$\overline{r^2} = \int_0^\infty r^2 dr \int_{-\infty}^\infty W(r,u) du.$$

$W(r,u)$ is the joint probability that a scatterer picked at random has a velocity component toward the transmitter in the interval du while returning a field having amplitude in dr. $G(\tau)$ is a complex function of τ with the demonstrable property

$$G(\tau) = G^*(-\tau) ; \qquad (4)$$

this property is a sufficient condition for the power spectrum of the complex amplitude to be real.

Of the various assumptions used by Siegert in deriving equation (3), several are of questionable validity when applied to weather targets. Although it is not the intention of this paper to assess the consequences of these assumptions, it should be mentioned that both theoretical and empirical evidence suggest that no significant errors arise from applying equation (3) to weather signals.

3. The power spectrum of the complex amplitude

The available information about the velocities of the scatterers is contained in the probability $W(r, u)$ which appears in equation (3) for the autocorrelation function. This probability is more simply related to the power spectrum of the complex amplitude than to its autocorrelation. The spectrum $F(f)$ is related to the autocorrelation by the transform pair

$$F(f) = \int_{-\infty}^{\infty} G(\tau) e^{-2\pi i f \tau} d\tau$$

$$G(\tau) = \int_{-\infty}^{\infty} F(f) e^{2\pi i f \tau} df .$$

The two-sided Fourier transform is used because $G(\tau)$, unlike the auto-correlation of real functions, is not an even function of τ. Substituting from (3) for $G(\tau)$ leads to

$$F(f) = \int_{-\infty}^{\infty} du \ S(u) \delta(\frac{2u}{\lambda} - f) = S(\frac{f\lambda}{2}), \tag{5}$$

where $\delta(f)$ is the Dirac delta function and

$$S(u) du = du \int_{0}^{\infty} \frac{r^2}{r^2} W(r, u) dr$$

is that fraction of the received power returned by scatterers whose velocity components in the direction of the transmitter lie in du. Since a frequency f corresponds, in the Doppler sense, to a velocity component $u = f\lambda/2$ toward the transmitter, the result (5) can be expressed

$$F(f) = S(u).\tag{6}$$

$F(f)$ is defined for positive and negative frequencies, corresponding respectively to velocities toward and away from the transmitter. Equation (6) shows that the power spectrum of the complex amplitude, henceforth called the Doppler spectrum, is an image of the velocity distribution of the scatterers, weighted according to the cross sections of the scatterers in each velocity interval. This simple relationship demonstrates the connection between the velocities of scatterers and the radar signal. It conveys the same information as equation (3) for the autocorrelation function, and can be used as the basis for interpreting signal fluctuations.

The signal sensed at the antenna is not the complex amplitude $e(t)$ but is the quantity $E(t)$ defined by (1). It is of interest, therefore, to determine how the spectrum of $E(t)$, which can be measured, is related to the Doppler spectrum $F(f)$ of the complex amplitude. This relation can be determined by first noting that

$$2E(t) = e(t)e^{-i\omega_0 t} + e^*(t)e^{i\omega_0 t}.$$

Then forming the product $E(t)E(t+\tau)$, taking time averages, and transforming to the frequency domain yield

$$\underline{F_m(f)} = \frac{1}{2}\left[F(-f-f_0) + F(f-f_0)\right].$$

Here $F_m(f) = \int_{-\infty}^{\infty} \frac{\overline{E(t)\,E(t+\tau)}}{\overline{E(t)^2}} e^{-2\pi i f \tau} d\tau$ is the power spectrum

of the received signal $E(t)$. The following normalizations hold for the spectra as defined above:

$$\int_{-\infty}^{\infty} F_m(f)\,df = \int_{-\infty}^{\infty} F(f)\,df = 1.$$

Power spectra are sometimes defined for positive values of frequency only, because the detection process used in measuring a spectrum makes no distinction between positive and negative frequencies of the same value. The measured spectrum $F_m(f)$ of $E(t)$ can therefore be defined as

$$F_m(f) \begin{cases} = F_m(f) + F_m(-f) = F(-f-f_0) + F(f-f_0), & f \geq 0 \\ = 0, & f < 0, \end{cases} \quad (7)$$

with the normalization

$$\int_0^{\infty} F_m(f)\,df = 1.$$

The spectrum $F_m(f)$ defined by (7) is available for measurement in experiments. The Doppler spectrum $F(f)$ from weather targets is restricted to an interval Δf, in the region of $f = 0$, which is very small compared to the carrier f_0. Consequently, $F(-f-f_0) = 0$ for all positive f, and equation (7) shows that $F_m(f)$ is simply the Doppler spectrum displaced along the frequency axis by an amount f_0. It is often inconvenient to spectrum-analyze a signal containing components as high as f_0. In this case $E(t)$ can be heterodyned down to a lower effective carrier frequency, say f', before being analyzed. Then the measured spectrum is given by (7) with f_0 replaced by f'. No difficulty arises from heterodyning the signal unless f' is chosen smaller than Δf, in which event the spectrum will have a contribution from the "folded" component $F(-f-f')$. Figure 1 illustrates this folding effect on the measured spectrum. The folding is most severe when the received signal is heterodyned against the transmitted signal.

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In this case $f' \doteq 0$ and any asymmetries in the Doppler spectrum are undetected. If, however, the velocity distribution and hence the Doppler spectrum were even functions, $F_m(f)$ would yet be sufficient information for describing the velocity distribution.

Transmitting in pulses introduces an additional consideration. If f_r is the pulse repetition frequency, the highest possible frequency in $F(f)$ is $\frac{1}{2} f_r$. Consequently, $F(f)$ will be an image of the velocity distribution only if no radial velocity components greater than $\pm f_r \lambda$ are present in the target.

We conclude that the power spectrum of the heterodyned signal can be used to calculate the radial velocity distribution of the scatterers. The receiver must be linear and coherent with the transmitter, all signal processing prior to spectrum analysis must be linear, and the final effective carrier frequency must be chosen large enough to prevent folding. Then the relation between the measured spectrum, the Doppler spectrum, and the velocity distribution is

$$ F_m(f) = F(f - f') = S\left(u - \frac{f'\lambda}{2} \right). \tag{8} $$

The Doppler spectrum $F(f)$, as we have defined it, is an intrinsic property of the radar signal independent of the way the signal is received and processed. When using Doppler radar, the quantity $F_m(f)$ can be measured and, as shown in (8), is closely related to the Doppler spectrum. With incoherent radar it is impossible, except in special cases, to calculate $F(f)$ (or correspondingly the velocity distribution) from any measured quantities. Certain properties of the velocity distribution can be derived from incoherent measurements, however, and it is convenient to describe these properties in terms of the Doppler spectrum.

4. Signal power

The power $I(t)$ in the signal described by (1) is proportional to $e(t)e^*(t) = A^2(t)$, and can be measured as the output of a square-law detector in the radar receiver. The power is the quantity ordinarily measured by incoherent weather radars. It is a fluctuating quantity because $A(t)$ fluctuates; the spectrum of its fluctuations is connected with the Doppler spectrum or $S(u)$, but in general cannot be used to obtain $S(u)$ because the phase information contained in $E(t)$ is lost in the process of forming $I(t)$. We will now consider how the loss of phase information restricts the data.

Siegert and Fleisher have both shown that the autocorrelation $g^2(\tau)$ of the power fluctuations is related to the autocorrelation of the complex amplitudes by

$$g^2(\tau) = G(\tau)\, G^*(\tau). \tag{9}$$

Of course $G(\tau)$ is generally complex, and it can be written so as to include the real function $g(\tau)$:

$$G(\tau) \equiv g(\tau)\, e^{i\psi(\tau)}$$

where $\psi(\tau)$ is also real. In order that the symmetric property of $G(\tau)$ be satisfied, namely $G(\tau) = G^*(-\tau)$, it is necessary and sufficient that $g(\tau)$ be even and $\psi(\tau)$ be odd.

The power spectrum $S_p(f)$ of power fluctuations is obtained by transforming the autocorrelation function. According to (9), equivalent expressions for the spectrum are

$$S_p(f) = \int_{-\infty}^{\infty} G(\tau)\, G^*(\tau)\, e^{-2\pi i f \tau}\, d\tau$$

$$S_p(f) = \int_{-\infty}^{\infty} g^2(\tau)\, e^{-2\pi i f \tau}\, d\tau.$$

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Both of these relationships can be written in terms of power spectra rather than correlation functions by noting

$$G(\tau) = \int_{-\infty}^{\infty} F'(f) e^{2\pi i f \tau} df$$

and by defining

$$g(\tau) = \int_{-\infty}^{\infty} F'(f) e^{2\pi i f \tau} df.$$

These substitutions lead to

$$S_p(f) = \int_{-\infty}^{\infty} F(\nu + f) F(\nu) d\nu = \int_{-\infty}^{\infty} F'(f - \nu) F'(\nu) d\nu \tag{10}$$

as equivalent expressions for the spectrum of power fluctuations. $F(f)$ is the Doppler spectrum while $F'(f)$ is an even function of frequency, obtained by transforming $g(\tau)$. The function $F'(f)$ can be computed from measurements with incoherent radar. It is of interest to determine under what circumstances $F'(f)$ can be used to obtain the velocity distribution.

Suppose the velocity distribution is an even function: that is, $S(u) = S(-u)$. Then $F(f) = F(-f)$ is true for the Doppler spectrum. This implies immediately that $G(\tau) = G(-\tau)$. We already know that $G(\tau) = G^*(-\tau)$, so we conclude

$$G(\tau) = G(-\tau) = G^*(\tau),$$

or $G(\tau)$ is a real function. Therefore, from (9),

$$G(\tau) = g(\tau).$$

In terms of the power spectra, this result becomes

$$F(f) = F'(f).$$

This shows that the Doppler spectrum is the same as the Fourier transform of the square root of the autocorrelation of power fluctuations when the velocity distribution is even. The velocity distribution can therefore be determined from incoherent measurements when it is an even function.

More generally, it can be verified that symmetry in the Doppler spectrum (or velocity distribution), rather than evenness, establishes a strong connection between the two spectrum functions. If $F(f)$ is symmetrical about f_i, that is, $F(f_i+f) = F(f_i-f)$, then

$$F(f_i+f) = F'(f).$$

This result, illustrated in Figure 2, means that the spectrum obtainable from incoherent measurements will have the same appearance as the Doppler spectrum when the velocity distribution is symmetrical. The mean velocity however, which corresponds to Doppler frequency f_1 , is lost in the incoherent measurement.

5. Level-crossings of the detected signal: the R-meter

A still more limited amount of information about the velocities of the scatterers is contained in the rate at which the power, or some function thereof, crosses an arbitrary level. Rutkowski and Fleisher (1955), for example, measured the rate at which the signal from a linear detector crossed its average value, and interpreted the results in terms of velocities. An instrument to measure this rate, called the R-meter, has been used by several investigators and is incorporated on some Weather Bureau research radars. Such measurements will now be shown to be related to certain properties of the Doppler spectrum.

Rice (1944) proved that the expected rate of zero crossings for Gaussian noise is uniquely determined by the spectrum of the noise. His analysis can be modified for application to signals that are not Gaussian, such as detected radar signals. The power returned are not Gaussian, such as detected radar signals. The power returned from random scatterers is distributed as the sum of the squares of two equal independent Gaussian random variables, as elaborated upon by Lawson and Uhlenbeck (1950). Using their

results in connection with the technique described by Rice leads to the following expression for the expected rate L at which the power crosses its average value:

$$L = \frac{\sqrt{8\pi}}{e} \left[\int_{-\infty}^{\infty} S_P(f) \, f^2 \, df \right]^{1/2},$$

where $S_P(f)$ is the power spectrum of power fluctuations. Substituting from (10) gives

$$L = \frac{\sqrt{8\pi}}{e} \left[\int_{-\infty}^{\infty} f^2 \, df \int_{-\infty}^{\infty} F(\nu + f) \, F(\nu) \, d\nu \right]^{1/2},$$

which reduces to

$$L = \frac{4\sqrt{\pi}}{e} \, \sigma_f \quad,$$

where σ_f is defined by

$$\sigma_f^2 = \overline{f^2} - \overline{f}^2 = \int_{-\infty}^{\infty} F(f) f^2 \, df - \left[\int_{-\infty}^{\infty} F(f) f \, df \right]^2.$$

In terms of the radial component u of velocity,

$$\sigma_f^2 = \frac{4}{\lambda^2} \left(\overline{u^2} - \overline{u}^2 \right) = \frac{4}{\lambda^2} \, \sigma_u^2.$$

Therefore, the mean rate at which the received power crosses its average level related to the standard deviation of the velocity distribution by

$$L = \frac{8\sqrt{\pi}}{\lambda e} \, \sigma_u = 5.22 \, \sigma_u / \lambda. \tag{11}$$

This relationship is applicable when the quantity measured is proportional to power, i.e. when a square law detector is used in the receiver. If a linear detector is used, the measured quantity is then proportional to $\left[I(t) \right]^{1/2}$ and the result of Rutkowski and Fleisher (1955) is applicable: the mean rate at which this signal crosses its average is

$$\frac{4\pi}{\lambda} e^{-\pi/4} \, \sigma_u = 5.74 \, \sigma_u / \lambda.$$

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Thus the proportionality factor which connects the standard deviation of velocity with the rate of average level crossings varies only about 10 per cent as the power law of the detector ranges from linear to square. Small deviations from an exact linear or square function in the detector are apparently of little importance in determining the proportionality factor.

Acknowledgements

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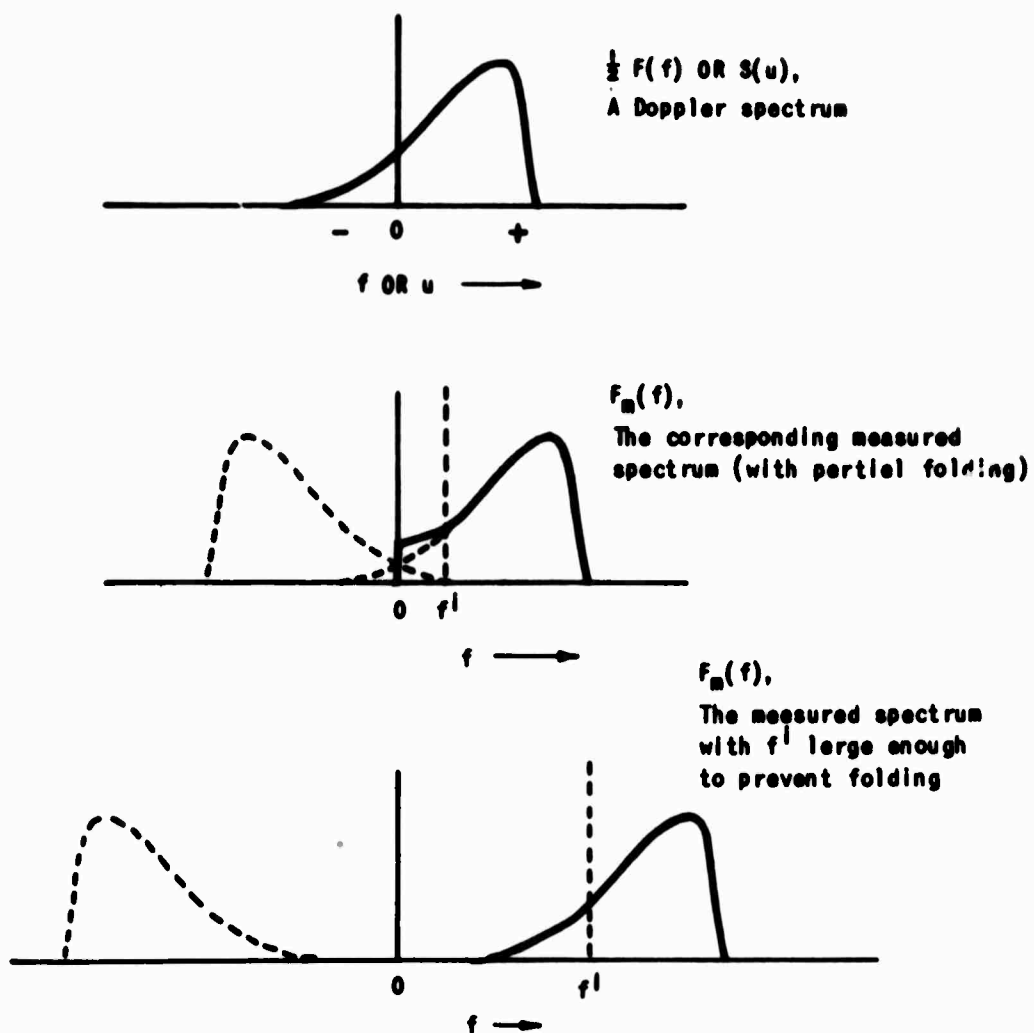
Figure 1 RELATION BETWEEN DOPPLER SPECTRUM AND MEASURED
SPECTRUM AFTER HETERODYNING

F(f) OR S(u),
A symmetrical
Doppler spectrum

f OR u ⟶

F'(f),
The corresponding spectrum
obtained by transforming g(t)

f ⟶
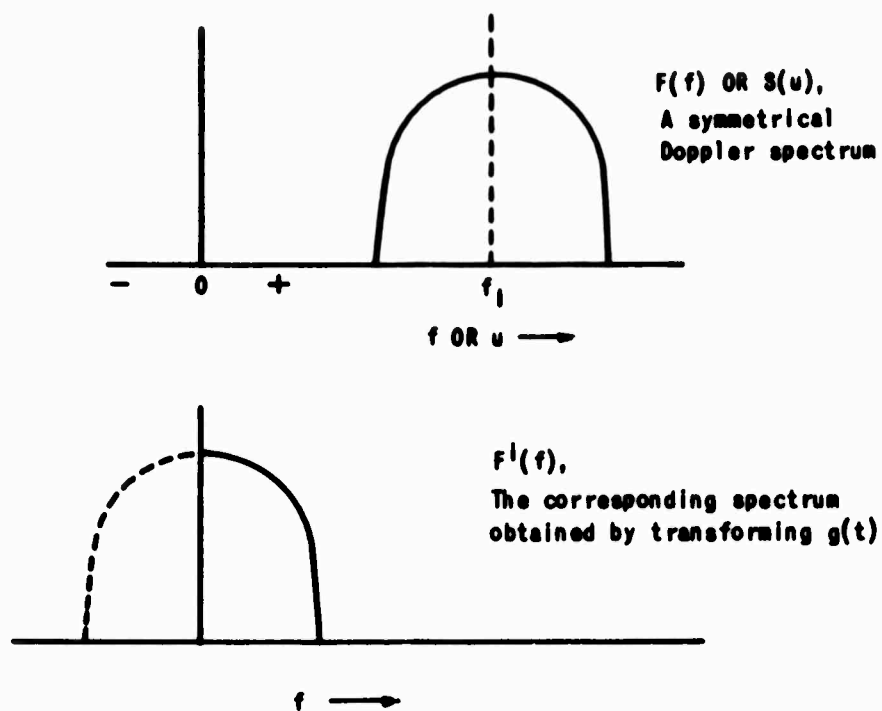
Figure 2 RELATION BETWEEN DOPPLER SPECTRUM AND SPECTRUM DERIVED FROM POWER MEASUREMENTS

APPENDIX B

Doppler Radar: A Probe for Atmospheric Turbulence

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Doppler Radar: A Probe for Atmospheric Turbulence

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ABSTRACT

From the time-varying Doppler spectrum of certain meteorological radar targets there can be derived the energy in the radial component of turbulence at target altitude. Two properties of the spectrum must be recorded continuously: the mean Doppler velocity $\langle V \rangle$ and the variance σ_v^2. Then the variance $\sigma_{\langle v \rangle}^2$ of the time-varying mean velocity and the average value $\overline{\sigma_v^2}$ of the variance must be computed from the record. The sum of these two computed quantities is proportional to the total energy E of the radial component of turbulence per unit mass of air. Specifically,

$$E = \frac{1}{2} \iiint_{-\infty}^{\infty} \Phi(\vec{k})\, d\vec{k} = \frac{1}{2}\left[\sigma_{\langle v \rangle}^2 + \overline{\sigma_v^2} \right],$$

where $\Phi(\vec{k})$ is the three-dimensional spectrum of the radial component of turbulence.

1.   Introduction

The radar signal backscattered from a weather target or a cloud of chaff can be processed to obtain information about the velocities of the particles comprising the target. Because of wind shear, turbulence, or unequal fall velocities, these particles are characterized by a velocity spread or distribution rather than by a single translational velocity. The distribution of the relative velocity of two particles chosen at random can be obtained by analyzing the fluctuations in power received by conventional noncoherent radars (Stone and Fleisher, 1956). For viewing conditions in which the effects of wind shear and unequal fall velocities can be estimated, the spread of the distribution of relative velocities has been used as a description of turbulence intensity, usually with the assumption that the illuminated particles respond instantly to wind changes (Rutkowski and Fleisher, 1955).

The scales of turbulence responsible for the spread of the velocity distribution are, in a rough sense, those smaller than the dimensions of the radar target volume. Larger scales of turbulence are not effectively manifested by spreading in the relative-velocity distribution, but rather by time variations in the distribution of particle velocities. The latter distribution, while not obtainable from measurements with noncoherent radars, can be simply derived from Doppler (coherent) radar measurements.

For those weather targets consisting of particles that respond faithfully to the wind, the velocity distribution of the particles can be interpreted as that of the wind. Therefore, since it measures the complete distribution of wind speed in a volume in space, Doppler radar can be considered as a volume-sampling anemometer. The purpose of this paper is to describe a way in which such an anemometer can be used to measure the total turbulent energy of the wind within any chosen volume of the atmosphere.

2.   Relation between Doppler Spectrum and Particle Velocity Distribution

The spectrum of the fluctuating signal amplitude received by a Doppler radar is closely related to the distribution of radial components of velocities of the scatterers comprising the radar target. The velocity distribution can

be described by the function S(V), where S(V)dV is the relative number of particles, weighted by their average radar cross sections, whose radial velocity components lie in the interval dV. This function is defined for positive and negative V, and is normalized to unit area. If the cross sections and velocities of the scatterers are uncorrelated, then the weighting is uniform and S(V) is the probability density of radial velocity.

If the output of a radar receiver which is linear, and coherent with the transmitter, is heterodyned down to frequency f' and then spectrum-analyzed, a measured spectrum $F_m(f)$ results. This spectrum is related to the weighted velocity distribution S(V) by

$$F_m(f) = \frac{1}{2}\left[S\left(\frac{f\lambda}{2} - \frac{f'\lambda}{2}\right) + S\left(-\frac{f\lambda}{2} - \frac{f'\lambda}{2}\right)\right] , \quad f \geq 0$$

$$= \ 0 \ \text{otherwise,}$$

where (f-f') $\lambda/2 = V$ is the usual relation between the Doppler shift (f-f'), carrier wavelength λ, and radial velocity V. The first term in the brackets would be expected from translation of the spectrum along the frequency axis as a result of heterodyning. The second term arises from the folding effect which occurs when the translated spectrum does not vanish at f = 0. These relations are illustrated in the Figure. We shall always choose f' large enough to prevent folding, in which case

$$F_m(f) \propto S\left(\frac{f\lambda}{2} - \frac{f'\lambda}{2}\right) ,$$

for positive f. Thus the measured Doppler spectrum is simply an image of the velocity distribution when frequencies are interpreted in the Doppler sense.

3.   Weather Targets and the Wind

For practical expedience in measuring the wind and its properties, we would like S(V) to be the same as the distribution of the radial component of wind in the radar-sampled volume. This wind has a mean and a turbulent component, both of which would ideally be represented by S(V). Stackpole (1961)

has shown that raindrops cannot be relied upon to respond well to gusts. Consequently, the measured S(V) from a target of raindrops will give only biased estimates of turbulence. Cloud droplets, on the other hand, are ideal wind tracers, but may not always backscatter enough energy to provide a detectable target for insensitive radars. Snowflakes lie somewhere between raindrops and cloud drops in terms of their responsiveness to wind and they are detectable at reasonable ranges by radars of modest capability. It has been found by extending Stackpole's analysis that dry snowflakes respond well enough to gusts to be considered adequate tracers for both the mean and the turbulent components of the wind. Additionally, their fall velocities are usually small – less than 1 m/sec – and the measured S(V) can be adjusted to account for those velocities. Although there is a correlation between the cross section and fall velocity of a snowflake, the range of vertical velocities is so small that no serious error is introduced by neglecting the weighting on S(V) due to cross section differences. Thus the wind distribution can easily be derived from the measured velocity distribution of snowflakes.[*]

Let us denote the wind distribution in the radar-sampled volume by S'(V). This function will differ noticeably from S(V) only for vertical viewing, when the full component of fall is detected. The properties of S'(V) which are important for our analysis are its mean value and the spread of velocities about that value. The effect of small scales of turbulence is to spread the distribution, while larger scales cause the location of the distribution on the V-axis to change with time. In application to turbulence measurements, the analysis is simplified if there are no small scale components of mean wind shear so that the spread of S'(V) is entirely due to turbulence; otherwise the contribution to spread due to mean wind shear must be subtracted from the total spread. The subtraction to account for mean shear would require an

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[*]Determination of the fidelity with which meteorological targets follow the wind requires lengthy computations which have not yet been carried out for typical radar chaff. Presumably they could be.

independent and precise measurement of the wind in the sampled volume, and such a measurement is not presently possible. Mean wind shear does not affect measurements for vertical viewing, however, and the small spread introduced into S(V) by the settling of snowflakes can easily be estimated and subtracted from the total spread to obtain S'(V). Therefore vertical measurements in snow constitute at least one situation in which the turbulent wind distribution can be obtained from the radar data.

4.   The Measurement of Turbulent Energy

As indicated above, S'(V), the distribution of radial components of turbulent velocity within the sampled volume, is readily obtained from S(V) for the case of vertical viewing. The time variations of the velocity variance and the average velocity can be obtained directly from S'(V) using analog techniques. We will now derive a fundamental relation between these quantities and the energy of the observed component of turbulence.

The (time) average value of the variance of the radial component of flow velocity $V(r)$ in the sampled volume v is defined by

$$\overline{\sigma_v^2(v)} = \frac{1}{v}\int_v V^2(\vec{r})\,dv - \overline{\left[\frac{1}{v}\int_v V(\vec{r})\,dv\right]^2}.$$

This variance is identical to the time average of the velocity variance of S'(V). Since time averages commute with space averages, the above equation can be rewritten as:

$$\overline{\sigma_v^2(v)} = \frac{1}{v}\int_v \overline{V^2(\vec{r})}\,dv - \frac{1}{v^2}\int_{v_1}dv_1\int_{v_2}dv_2\,\overline{V(\vec{r_1})V(\vec{r_2})}$$

$$= \overline{V^2(\vec{r})} - \frac{1}{v^2}\int_{v_1}dv_1\int_{v_2}dv_2\,B_V(\vec{r_1}-\vec{r_2}),$$

where $B_V(\vec{r}_1 - \vec{r}_2)$ is the three-dimensional correlation function[*] of $V(\vec{r})$,

[*] See Tatarski, Wave Propagation in a Turbulent Medium, McGraw-Hill, 1961, for a discussion of the functions used in describing random fields.

and time averages have been replaced by ensemble averages. If the field $V(\vec{r})$ is assumed to be homogeneous, the three-dimensional spectrum $\tilde{\Phi}(\vec{k})$ of V can be introduced, giving

$$\overline{\sigma_v^2(w)} = \iiint\limits_{-\infty}^{\infty} \tilde{\Phi}(\vec{k})\left[1 - f(w; \vec{k})\right] d\vec{k}, \qquad (1)$$

where $\vec{k}$ is the wave vector and $f(v; \vec{k})$ is a function determined by the size and shape of the radar-sampled volume. For any volume, this function equals unity at $k = 0$ and approaches zero as k increases. Thus $\overline{\sigma_v^2(w)}$ is more sensitive to large wave numbers than to small ones. For ordinary radar target-volumes, the result is, roughly, that the average variance of the velocity distribution is sensitive to scales of turbulence smaller than a characteristic length of the volume, and relatively insensitive to scales larger than this length.

The other property of S'(V) related to the turbulent energy is the variance in the time domain of the average value $\langle V \rangle$ of velocity over the distribution. A development similar to that above shows this variance to be

$$\sigma_{\langle v \rangle}^2(w) = \iiint\limits_{-\infty}^{\infty} \tilde{\Phi}(\vec{k}) f(w; \vec{k}) d\vec{k}, \qquad (2)$$

where $f(v; \vec{k})$ is the form function previously described. A comparison of (1) and (2) shows that $\sigma_{\langle v \rangle}^2(w)$ is sensitive to the form function $f(v; \vec{k})$ in a manner exactly complementary to $\overline{\sigma_v^2(w)}$; i.e., the quantity defined by (2) is sensitive to the large scales of turbulence, and relatively insensitive to the small scales. As already indicated, the converse is true for $\overline{\sigma_v^2(w)}$. Indeed, the relative magnitudes of $\sigma_{\langle v \rangle}^2(w)$ and $\overline{\sigma_v^2(w)}$ indicate the partitioning of the turbulent energy between turbulent scales larger than and smaller than the sampled volume.

Adding (1) and (2), we obtain a result independent of the sampled volume:

$$\sigma_{\langle v \rangle}^2(w) + \overline{\sigma_v^2(w)} = \iiint\limits_{-\infty}^{\infty} \tilde{\Phi}(\vec{k}) d\vec{k}. \qquad (3)$$

The term on the right is equal to twice the energy E, per unit mass of air, in the measured component V of turbulence. Consequently, the energy is related to the computed quantities by

$$E = \frac{1}{2}\left[\sigma_{\langle v \rangle}^2(w) + \overline{\sigma_v^2(w)}\right]. \qquad (4)$$

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The total turbulent energy E_T (in all components) can be obtained only with the assumption of isotropy, in which case

$$E_T = \frac{3}{2}\left[\sigma_{(v)}^{2}(\nu) + \overline{\sigma_v^2(\nu)}\right].$$

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$s(v)$
a velocity distribution

$F_m(f)$,
the corresponding measured Doppler
spectrum (with partial folding)

$F_m(f)$,
the measured spectrum
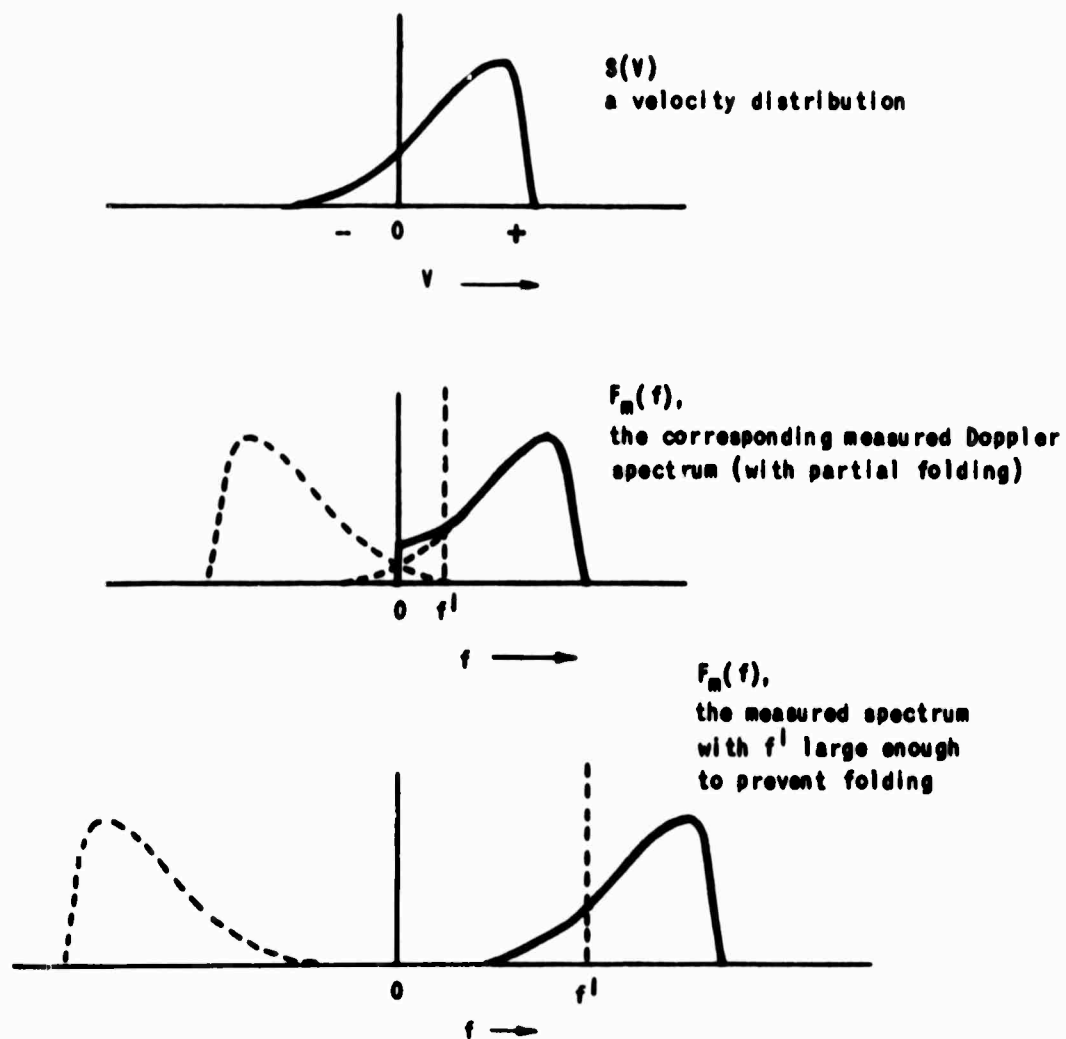with f^l large enough
to prevent folding

Figure 1 RELATION BETWEEN VELOCITY DISTRIBUTION OF SCATTERERS
AND MEASURED SPECTRUM AFTER HETERODYNING

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